NJIT New Jersey Institute of Technology | Center for Pre-College Programs | Campbell Hall, 4th & 5th Floors | University Heights Newark, New Jersey 07102-1982 Phone: 973-596-3550 njit.edu/precollege

Helping Kids Learn – Post #16 5/15/20

STEAM: Art & Mathematics

Math is known as the *Queen of the Sciences.* It could equally well claim the title *Empress of the Arts*. Math is imbedded in everything from music to drawing and painting to CGI in movies. This exploration can be adapted for use by young children to adults. See **Lift the Level** below.

Be an Artist & Mathematician – Exploring Transformations

A plane in geometry is a flat surface. It has two dimensions – length and width. **Transformations** are specific ways to move shapes in the plane. They change the place of a shape something like the Transformer toy changes its shape. The shape of the shape and its perimeter and area stay the same:

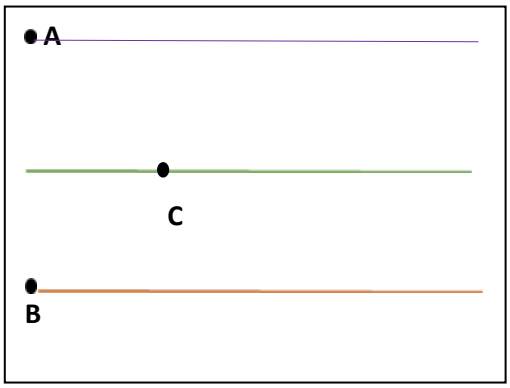
- **Translation**
- **Rotation**
- **Reflection**

In Investigations 1, 2 and 3, you will explore each transformation.

A vertex is where two sides meet, a corner. "Vertices" is the plural of "vertex."

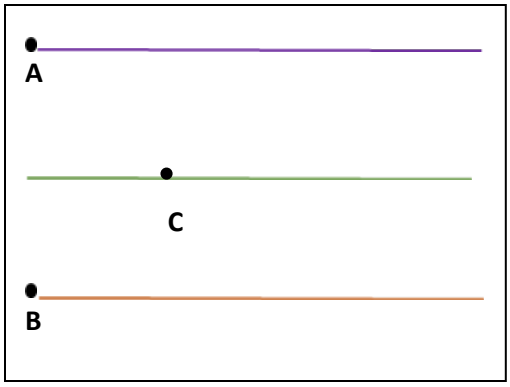
Investigation 1: A **translation** is just a slide. Cut out Triangle **ABC** on page **X.** Match the vertices of the triangle to the points **A, B, C** here. Trace around the triangle so you can see where Triangle ABC was.

Slide Triangle ABC without picking it up. Keep the vertices on the purple, green and orange lines. Trace around the second triangle and label the new vertices **A', B', C'** ("A prime, B prime, C prime").



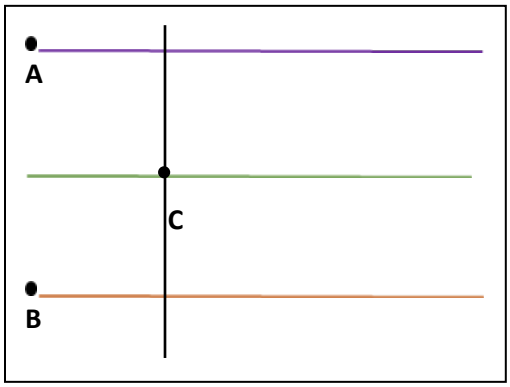
Mathematicians use "prime" to name the vertices instead of all new letters because the two triangles are identical except for position. (See **Solutions/Suggestions** below for suggested answers.)

Investigation 2: A **rotation** is a turn or spin. Use Triangle **ABC** again. Place the vertices on top of A, B, and C, respectively and trace. Keep the triangle flat. This time, turn it (pivot) around vertex **C.** Trace the new triangle and label the vertices A', B' and C'. Regardless of where A' and B' are on your picture, C' should be the same point as C.



(See **Solutions/Suggestions**.)

Investigation 3: A **reflection** is a flip across a line in the plane. You see your *reflection* in a mirror. Think of the line as a mirror. In Investigations 1 and 2 you got to make some choices about where to put Triangle A'B'C'. Not so with a reflection. Use the black line as your line of reflection. If you imagine a mirror exactly vertical on that line, there's only one place to put Triangle A'B'C'. Trace Triangle ABC, reflect it across the black line. Trace Triangle A'B'C'. As in Investigation 2, point C and point C' are the same.



Investigation 4: So where's the art in all this math? Many artists use the concepts of **transformational geometry** in their work. One painter who did so many times was M.C. Escher. Here's an example of a **translation** of a drawing of a bird.

Escher catalog #128.

Cut out the whole black bird on the **Worksheet**. (Keep the white one in case you want to do more with Escher). Place it on top of the black bird at the top left in this picture. Then translate it to be exactly on top of the white bird to the right. Repeat going right.

Start with your bird on the top left black bird again. Translate to cover the white bird below it. Repeat going down the picture.

Start with your bird on any bird in the picture. Translate along a diagonal line (the birds will be the same color as the one you start with).

Reflect the bird along the horizontal line on the Worksheet. Make your own picture by tracing and translating the bird. You can use different color combinations or give the bird different details. How does your drawing compare to Escher's?

Lift the Level! You can make this lesson deeper and/or suitable for older students by any of the following. You may find links in **STEAM Online** useful:

1. This can be done by any age group. Create your own, original transformational drawings as follows: cut out the square on the Worksheet. Draw a line from top to bottom in any way you wish. Cut carefully along the line. Lay the two pieces on a flat surface so they form the original square. Slide (translate) the right-hand piece to the left so the (former) opposite sides of the square meet exactly. Tape along that joint. Lay your new shape on a piece of blank paper. Trace. Then translate or rotate or reflect it and trace again. Repeat several times. Step back and analyze your drawing. What do your shapes look like? Escher "saw" animals and other objects in the shapes he created so he gave them features like eyes and noses. Give your drawing features or designs and color them in.

2. Placing two-dimensional shapes on a grid, or Cartesian plane, allows mathematicians to further formalize the transformations and connect them to algebra. Use Triangle ABC from the Investigations and the grid on the Worksheet. Place vertex A at the origin (0, 0). Mark and label points B and C. Then rotate 180° around point A. Mark and label points B' and C' (A' and A are coincident). Write an equation to describe this transformation. (See **Solutions/Suggestions**.)
3. The transformations in this Exploration are known as **rigid transformations** because shapes remain rigid (points map onto transformed points described by linear equations.) If you deform the original shape and transform the deformation, it is a **skew transformation**. Think about a funhouse mirror. Artists such as Jan van Eyck and Salvador Dali have used skew transformations in their work. Look at examples online and create a poster describing the math behind one artistic skew transformation.
4. M.C. Escher used many forms of transformational geometry in his work as well as color symmetries. He has an enormous, and enormously complex, body of work. Use the links in **STEAM Online** to explore his rotations and skew transformations. Make a visual display explaining the transformations Escher used.

STEAM Online

These are suggestions only and no endorsement is implied.

Although they have been screened for appropriateness before posting, parents should vet the websites their children use as they may change over time.

Transformations	https://www.mathsisfun.com/geometry/transformations.html http://www.gradeamathhelp.com/transformation-geometry.html
M.C. Escher	https://mcescher.com/gallery/ especially https://mcescher.com/gallery/symmetry/ https://en.wikipedia.org/wiki/M._C._Escher
Jan van Eyck	https://en.wikipedia.org/wiki/Jan_van_Eyck especially https://en.wikipedia.org/wiki/Arnolfini_Portrait
Salvador Dali	http://www.salvadordali.com/ especially http://www.salvadordali.com/featured/dali1/

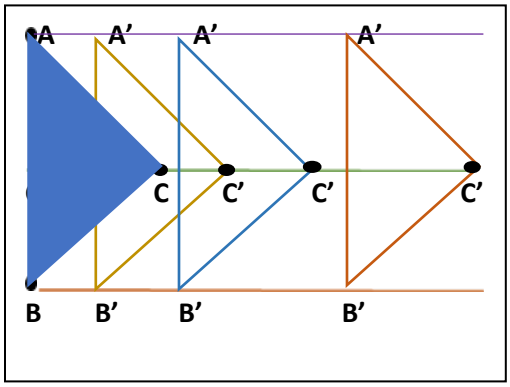
NJ Student Learning Standards

Mathematics Standards K.G.B.4, K.G.B.5; 1.G.A.1; 2.G.A.1; 3.G.C.5; 4.G.A.3; 5.G.A; 6.G.A; 7.G.B; 8.G.A; HS-C-GO.A.2, HS-C-G0.A.5.

Visual and Performing Arts Standards 1.1.2.D.1; 1.1.5.D.1; 1.1.8.D.2; 1.1.12.D.1

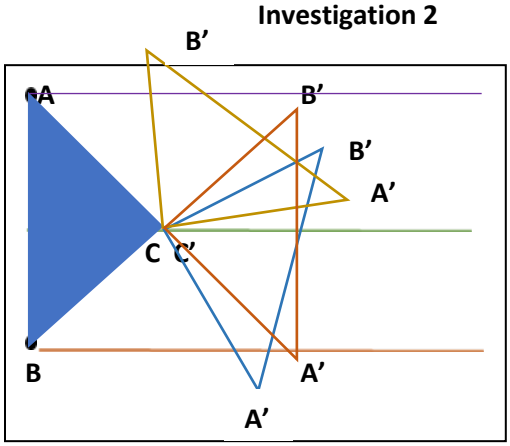
Solutions/Suggestions

Investigation 1



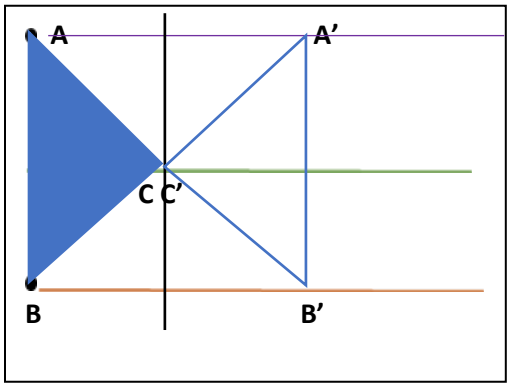
As long as the vertices touch the purple, green, and orange lines and the orientation of the triangle stays the same, any position is OK. Three of an infinite number of triangles are shown in gold, blue and orange. The order of the vertices must stay the same as the original triangle.

Investigation 2



As long as point C' is exactly on top of point C, you can spin and stop the triangle anywhere. Notice that A' seems to have switched places with B and B' with A. If you follow the vertices around the rotation, you'll know that is not the case.

Investigation 3

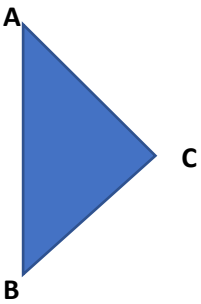


A reflection is a flip across a line. Because C was touching the line, C' must touch it, also, at the same point. When the triangle is flipped across a vertical line, the other two vertices stay in relatively the same positions so A maps to A' and B maps to B'.

Lift the Level #2 T(A, B) = T(-A, -B) or, more generally, T(*x, y*) = T(*-x, -y*) where *x, y* is any point on the triangle.

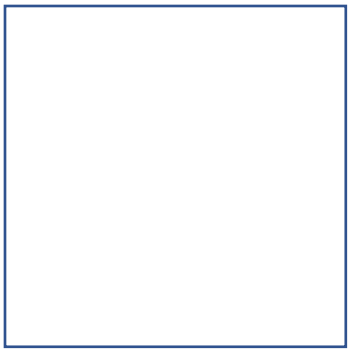
Worksheet

For **Investigations** 1, 2, and 3

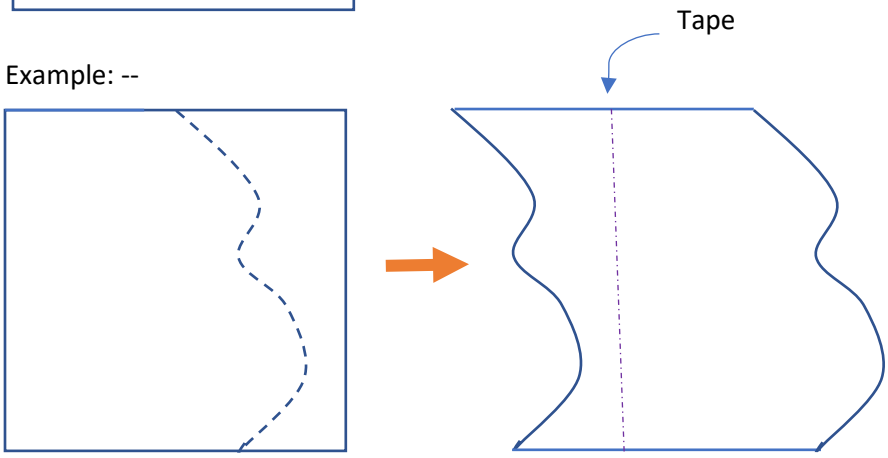


For **Investigation** 4

For **Lift the Level** #1



Example: --



For **Lift the Level** #2